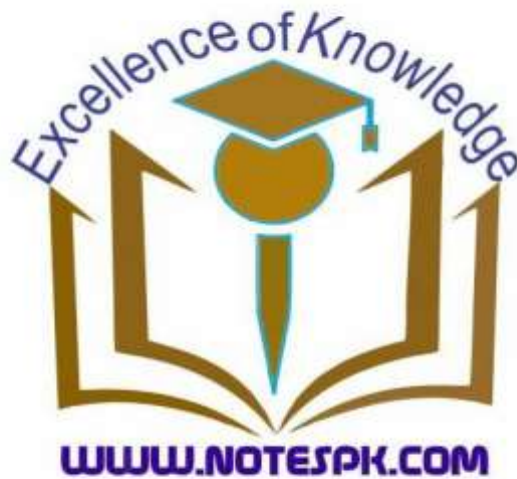


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# Chapter 9.

## INTRODUCTION TO CO-ORDINATES GEOMETRY



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## Coordinate Geometry

The study of geometrical shapes in a plane is called plane geometry. Coordinate geometry is the study of geometrical shapes in the Cartesian plane (coordinate plane).

### Distance Formula

Let  $P(x_1, y_1)$  and  $Q(x_2, y_2)$  be two points in the coordinate plane where  $d$  is the length of the line segment  $PQ$ . i.e.  $|PQ| = d$  and given as

$$|d| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

### EXERCISE 9.1

Q#1) Find the distance between the following pairs of points.

(a)  $A(9, 2), B(7, 2)$

Sol: As given  $A(9, 2), B(7, 2)$

Using distance formula

$$|d| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Put  $x_1 = 9, x_2 = 7, y_1 = 2$  and  $y_2 = 2$

$$|d| = \sqrt{(7 - 9)^2 + (2 - 2)^2}$$

$$|d| = \sqrt{(-2)^2 + (0)^2}$$

$$|d| = \sqrt{4}$$

$$|d| = 2$$

(b)  $A(2, -6), B(3, -6)$

Sol: As given  $A(2, -6), B(3, -6)$

Using distance formula

$$|d| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Put  $x_1 = 2, x_2 = 3, y_1 = -6$  and  $y_2 = -6$

$$|d| = \sqrt{(3 - 2)^2 + (-6 - (-6))^2}$$

$$|d| = \sqrt{(1)^2 + (0)^2}$$

$$|d| = \sqrt{1}$$

$$|d| = 1$$

(c)  $A(-8, 1), B(6, 1)$

Sol: As given  $A(-8, 1), B(6, 1)$

Using distance formula

$$|d| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Put  $x_1 = -8, x_2 = 6, y_1 = 1$  and  $y_2 = 1$

$$|d| = \sqrt{(6 - (-8))^2 + (1 - 1)^2}$$

$$|d| = \sqrt{(6 + 8)^2 + (0)^2}$$

$$|d| = \sqrt{14^2}$$

$$|d| = 14$$

(d)  $A(-4, \sqrt{2}), B(-4, -3)$

Sol: As given  $A(-4, \sqrt{2}), B(-4, -3)$

Using distance formula

$$|d| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Put  $x_1 = -4, x_2 = -4, y_1 = \sqrt{2}$  and  $y_2 = -3$

$$|d| = \sqrt{(-4 - (-4))^2 + (-3 - \sqrt{2})^2}$$

$$|d| = \sqrt{(-4 + 4)^2 + (-3 - \sqrt{2})^2}$$

$$|d| = \sqrt{(3 + \sqrt{2})^2}$$

$$|d| = 3 + \sqrt{2}$$

(e)  $A(3, -11), B(3, -4)$

Sol: As given  $A(3, -11), B(3, -4)$

Using distance formula

$$|d| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Put  $x_1 = 3, x_2 = 3, y_1 = -11$  and  $y_2 = -4$

$$|d| = \sqrt{(3 - 3)^2 + (-4 - (-11))^2}$$

$$|d| = \sqrt{(0)^2 + (-4 + 11)^2}$$

$$|d| = \sqrt{(7)^2}$$

$$|d| = 7$$

(f)  $A(0, 0), B(0, -5)$

Sol: As given  $A(0, 0), B(0, -5)$

Using distance formula

$$|d| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Put  $x_1 = 0, x_2 = 0, y_1 = 0$  and  $y_2 = -5$

$$|d| = \sqrt{(0 - 0)^2 + (-5 - 0)^2}$$

$$|d| = \sqrt{(0)^2 + (-5)^2}$$

$$|d| = \sqrt{5^2}$$

$$|d| = 5$$

Q#2) Let P be the point on  $x$ -axis with  $x$ -coordinate  $a$  and Q be the point on  $y$ -axis with  $y$ -coordinate  $b$  as given below. Find the distance between P and Q.

(i)  $a = 9, b = 7$

Sol: As Given  $a = 9, b = 7$

$$P(a, 0) = P(9, 0) \text{ and } Q(0, b) = Q(0, 7)$$

Using distance formula

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|PQ| = \sqrt{(0 - 9)^2 + (7 - 0)^2}$$

$$|PQ| = \sqrt{(-9)^2 + (7)^2}$$

$$|PQ| = \sqrt{81 + 49}$$

$$|PQ| = \sqrt{130}$$

(ii)  $a = 2, b = 3$

Sol: As Given  $a = 2, b = 3$

$$P(a, 0) = P(2, 0) \text{ and } Q(0, b) = Q(0, 3)$$

Using distance formula

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|PQ| = \sqrt{(0 - 2)^2 + (3 - 0)^2}$$

$$|PQ| = \sqrt{(-2)^2 + (3)^2}$$

$$|PQ| = \sqrt{4 + 9}$$

$$|PQ| = \sqrt{13}$$

(iii)  $a = -8, b = 6$

Sol: As Given  $a = -8, b = 6$

$$P(a, 0) = P(-8, 0) \text{ and } Q(0, b) = Q(0, 6)$$

Using distance formula

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|PQ| = \sqrt{(0 - (-8))^2 + (6 - 0)^2}$$

$$|PQ| = \sqrt{(8)^2 + (6)^2}$$

$$|PQ| = \sqrt{64 + 36}$$

$$|PQ| = \sqrt{100} = 10$$

(iv)  $a = -2, b = -3$

Sol: As Given  $a = -2, b = -3$

$$P(a, 0) = P(-2, 0) \text{ and } Q(0, b) = Q(0, -3)$$

Using distance formula

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|PQ| = \sqrt{(0 - (-2))^2 + (-3 - 0)^2}$$

$$|PQ| = \sqrt{(2)^2 + (-3)^2}$$

$$|PQ| = \sqrt{4 + 9}$$

$$|PQ| = \sqrt{13}$$

(v)  $a = \sqrt{2}, b = 1$

Sol: As Given  $a = \sqrt{2}, b = 1$

$$P(a, 0) = P(\sqrt{2}, 0) \text{ and } Q(0, b) = Q(0, 1)$$

Using distance formula

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|PQ| = \sqrt{(0 - \sqrt{2})^2 + (1 - 0)^2}$$

$$|PQ| = \sqrt{(\sqrt{2})^2 + (1)^2}$$

$$|PQ| = \sqrt{2 + 1}$$

$$|PQ| = \sqrt{3}$$

(vi)  $a = -9, b = -4$

Sol: As Given  $a = -9, b = -4$

$$P(a, 0) = P(-9, 0) \text{ and } Q(0, b) = Q(0, -4)$$

Using distance formula

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|PQ| = \sqrt{(0 - (-9))^2 + (-4 - 0)^2}$$

$$|PQ| = \sqrt{(9)^2 + (-4)^2}$$

$$|PQ| = \sqrt{81 + 16}$$

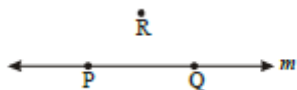
$$|PQ| = \sqrt{97}$$

## Collinear or Non-collinear Points in the Plane

Two or more than two points which lie on the same straight line are called collinear points with respect to that line; otherwise they are called non-collinear.

Let  $m$  be a line, then all the points on line  $m$  are collinear.

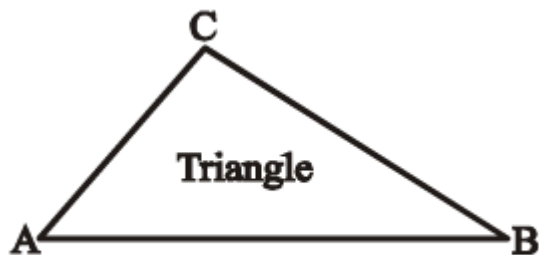
In the given figure, the points P and Q are collinear with respect to the line  $m$  and the points P and R are not collinear with respect to it.



## Triangle

A closed figure in a plane obtained by joining three non-collinear points is called a triangle.

In the triangle  $ABC$  the non-collinear points  $A, B$  and  $C$  are the three vertices of the triangle  $ABC$ . The line segments  $AB, BC$  and  $CA$  are called sides of the triangle.



### (i) Equilateral Triangle

If the lengths of all the three sides of a triangle are same, then the triangle is called an equilateral triangle.

### (ii) An Isosceles Triangle

An isosceles triangle  $PQR$  is a triangle which has two of its sides with equal length while the third side has a different length.

### (iii) Right Angle Triangle

A triangle in which one of the angles has measure equal to  $90^\circ$  is called a right angle triangle.

### (iv) Scalene Triangle

A triangle is called a scalene triangle if measures of all the three sides are different.

## Square

A square is a closed figure in the plane formed by four non-collinear points such that lengths of all sides are equal and measure of each angle is  $90^\circ$ .

## Rectangle

A figure formed in the plane by four non-collinear points is called a rectangle if,

- (i) Its opposite sides are equal in length;
- (ii) The angle at each vertex is of measure  $90^\circ$ .

## Parallelogram

A figure formed by four non-collinear points in the plane is called a **parallelogram** if

- (i) its opposite sides are of equal length
- (ii) its opposite sides are parallel

Let  $P(x_1, y_1)$  and  $Q(x_2, y_2)$  be any two points in the plane and  $M(x, y)$  be a mid-point of points  $P$  and  $Q$  on the line-segment  $PQ$  is given as

$$\text{Mid - point of } PQ = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

## EXERCISE 9.3

Q#1) Find the mid-point of the line segment joining each of the following pairs of points

- (a)  $A(9, 2), B(7, 2)$

Sol: As given  $A(9, 2), B(7, 2)$

Using Mid-point formula

$$\text{Mid - point of } PQ = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Put  $x_1 = 9, x_2 = 7, y_1 = 2$  and  $y_2 = 2$

$$\text{Mid - point of } PQ = M\left(\frac{9 + 7}{2}, \frac{2 + 2}{2}\right)$$

$$\text{Mid - point of } PQ = M\left(\frac{16}{2}, \frac{4}{2}\right)$$

$$\text{Mid - point of } PQ = M(8, 2)$$

- (b)  $A(2, -6), B(3, -6)$

Sol: As given  $A(2, -6), B(3, -6)$

Using Mid-point formula

$$\text{Mid - point of } PQ = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Put  $x_1 = 2, x_2 = 3, y_1 = -6$  and  $y_2 = -6$

$$\text{Mid - point of } PQ = M\left(\frac{2 + 3}{2}, \frac{-6 - 6}{2}\right)$$

$$\text{Mid - point of } PQ = M\left(\frac{5}{2}, \frac{-12}{2}\right)$$

$$\text{Mid - point of } PQ = M(2.5, -6)$$

- (c)  $A(-8, 1), B(6, 1)$

Sol: As given  $A(-8, 1), B(6, 1)$

Using Mid-point formula

$$\text{Mid - point of } PQ = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Put  $x_1 = -8, x_2 = 6, y_1 = 1$  and  $y_2 = 1$

$$\text{Mid - point of } PQ = M\left(\frac{-8+6}{2}, \frac{1+1}{2}\right)$$

$$\text{Mid - point of } PQ = M\left(\frac{-2}{2}, \frac{2}{2}\right)$$

$$\text{Mid - point of } PQ = M(-1, 1)$$

(d)  $A(-4, 9), B(-4, -3)$

Sol: As given  $A(-4, \sqrt{2}), B(-4, -3)$

Using Mid-point formula

$$\text{Mid - point of } PQ = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Put  $x_1 = -4, x_2 = -4, y_1 = 9$  and  $y_2 = -3$

$$\text{Mid - point of } PQ = M\left(\frac{-4-4}{2}, \frac{9-3}{2}\right)$$

$$\text{Mid - point of } PQ = M\left(\frac{-8}{2}, \frac{6}{2}\right)$$

$$\text{Mid - point of } PQ = M(-4, 3)$$

(e)  $A(3, -11), B(3, -4)$

Sol: As given  $A(3, -11), B(3, -4)$

Using Mid-point formula

$$\text{Mid - point of } PQ = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Put  $x_1 = 3, x_2 = 3, y_1 = -11$  and  $y_2 = -4$

$$\text{Mid - point of } PQ = M\left(\frac{3+3}{2}, \frac{-11-4}{2}\right)$$

$$\text{Mid - point of } PQ = M\left(\frac{6}{2}, \frac{-15}{2}\right)$$

$$\text{Mid - point of } PQ = M(3, -7.5)$$

(f)  $A(0, 0), B(0, -5)$

Sol: As given  $A(0, 0), B(0, -5)$

Using Mid-point formula

$$\text{Mid - point of } PQ = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Put  $x_1 = 0, x_2 = 0, y_1 = 0$  and  $y_2 = -5$

$$\text{Mid - point of } PQ = M\left(\frac{0+0}{2}, \frac{0-5}{2}\right)$$

$$\text{Mid - point of } PQ = M\left(\frac{0}{2}, \frac{-5}{2}\right)$$

$$\text{Mid - point of } PQ = M(0, 2.5)$$

## REVIEW EXERCISE

Q#2) Answer the following, which is true and which is false.

(i) A line has two end points... **F** .....

(ii) A line segment has one end point... **F** .....

(iii) A triangle is formed by three collinear points. ... **F**

(iv) Each side of a triangle has two collinear vertices... **T**...

(v) The end points of each side of a rectangle are collinear... **T** ....

(vi) All the points that lie on the x-axis are collinear... **T** ...

(vii) Origin is the only point collinear with the points of both the axes separately. ... **T**..